ENGINEERING ECONOMY

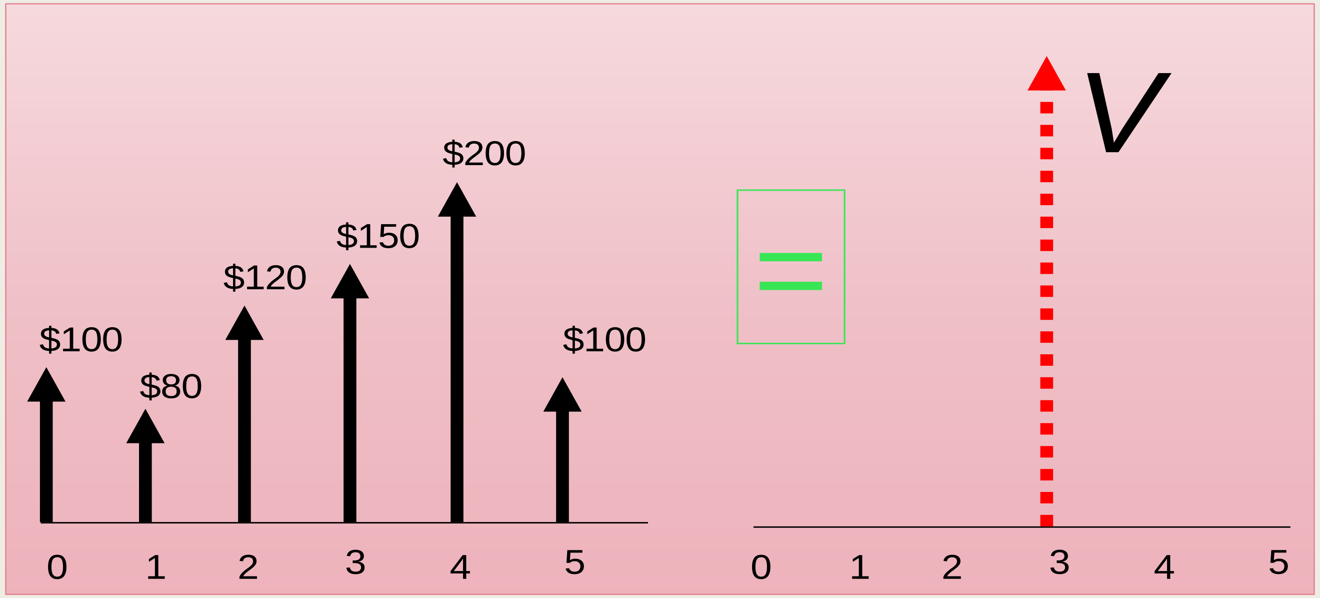
Time Value of Money (2)

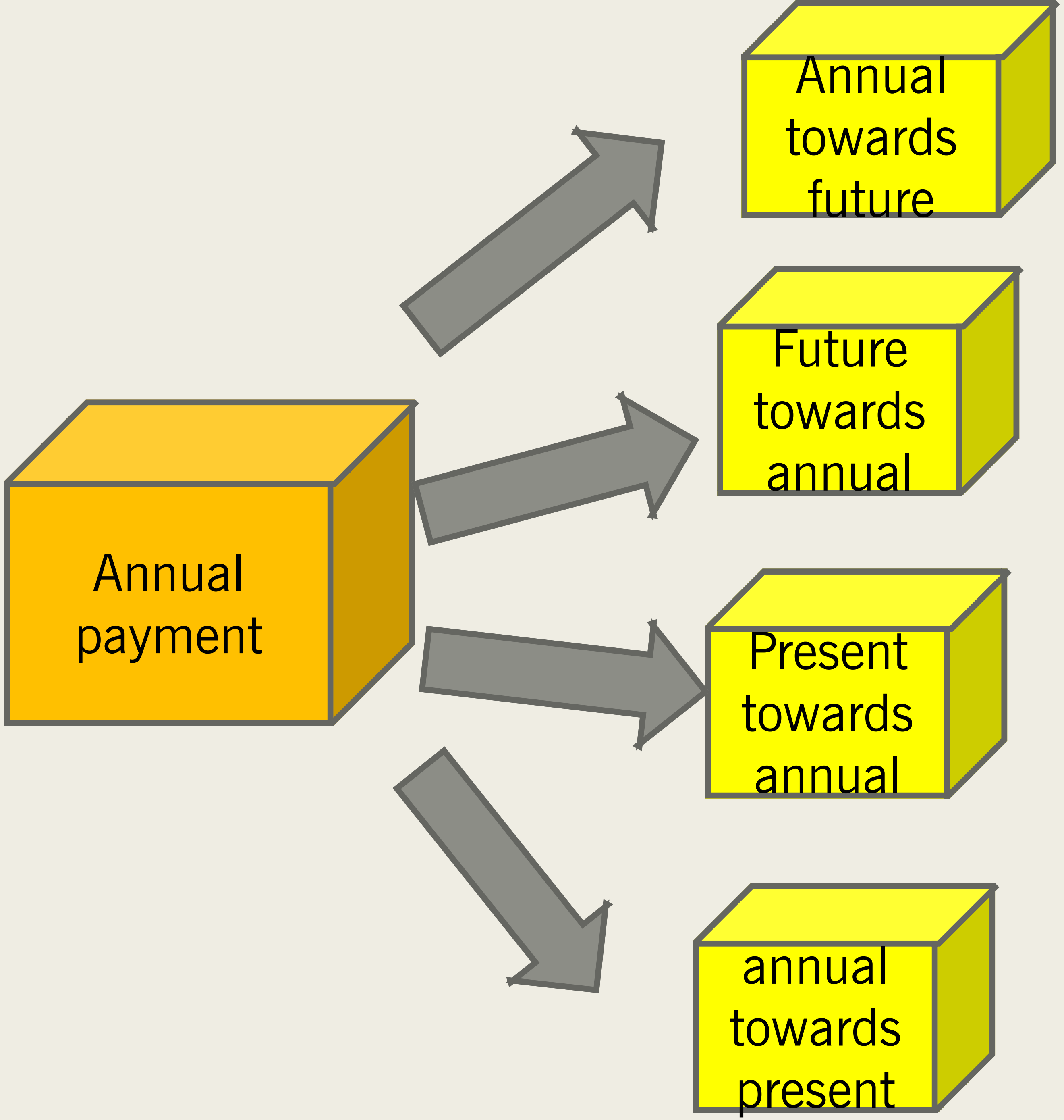
LETS DO SUCH WARMING UP!

Are you ready?

Practice Problem 1

Compute the equivalent value of the cash flow series at $n = 3$, using $i = 9\%$.




Annual payment
Annual towards future
Future towards annual
Present towards annual
annual towards present

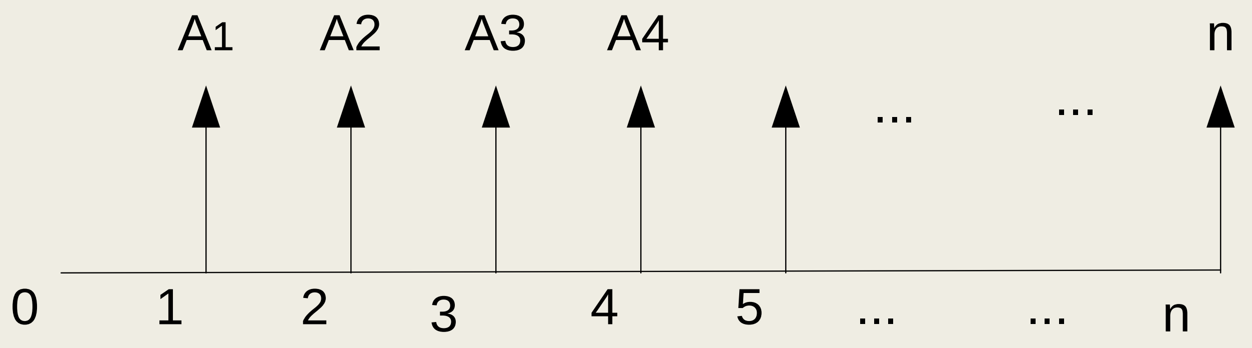
2. **Cash Flow Annual**

$$F = A.\left(\frac{(1+i)^n - 1}{i}\right) \rightarrow F = A(F/A,i,n)$$

$$\left(\frac{(1+i)^n - 1}{i}\right)$$: *uniform series compound amount factor*

$$A = F.\left(\frac{i}{(1+i)^n - 1}\right) \rightarrow A = F(A/F,i,n)$$

$$\left(\frac{i}{(1+i)^n - 1}\right)$$: *uniform series sinking fund factor*



Annual Cash Flow

$$A = P.\left(\frac{i.(1+i)^n}{(1+i)^n - 1}\right) \rightarrow A = P(A/P,i,n)$$

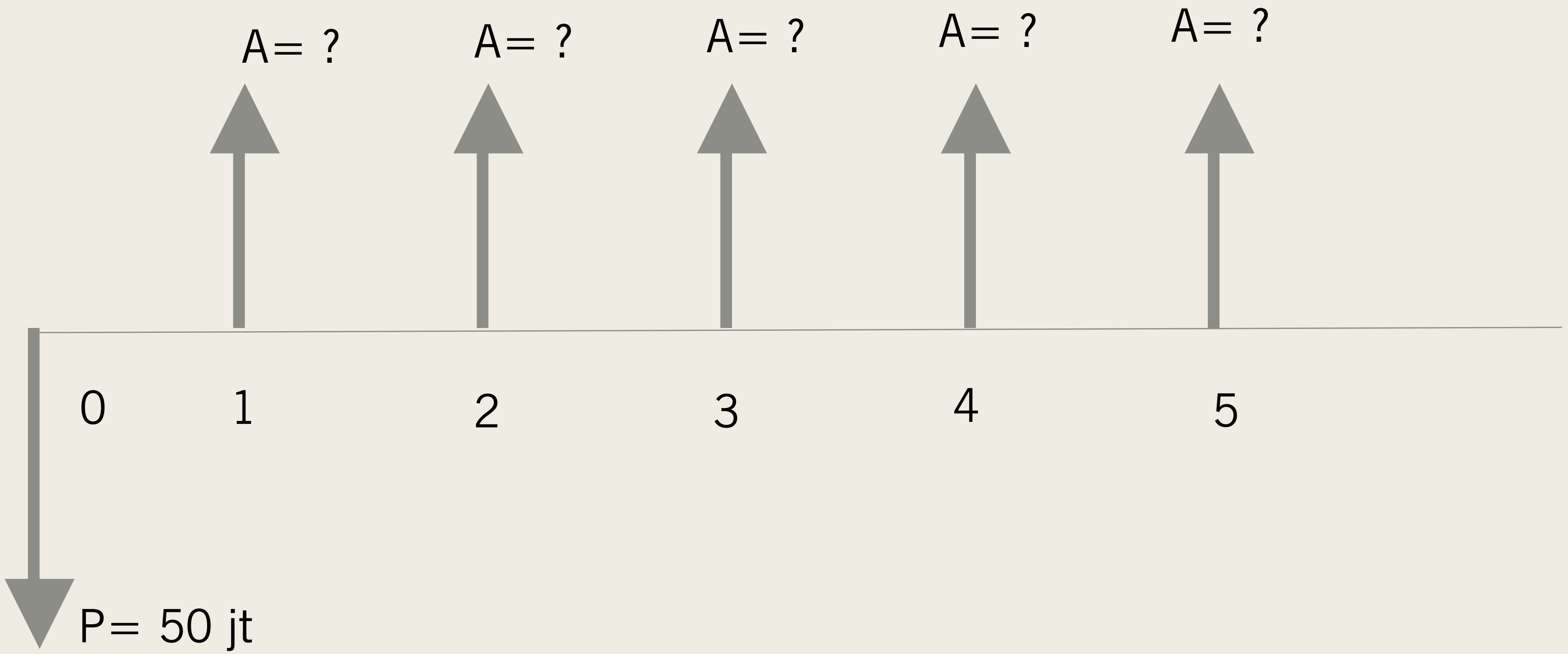
$$\left(\frac{i.(1+i)^n}{(1+i)^n - 1}\right)$$: *uniform series capital recovery factor*

$$P = A.\left(\frac{(1+i)^n - 1}{i.(1+i)^n}\right) \rightarrow P = A(P/A,i,n)$$

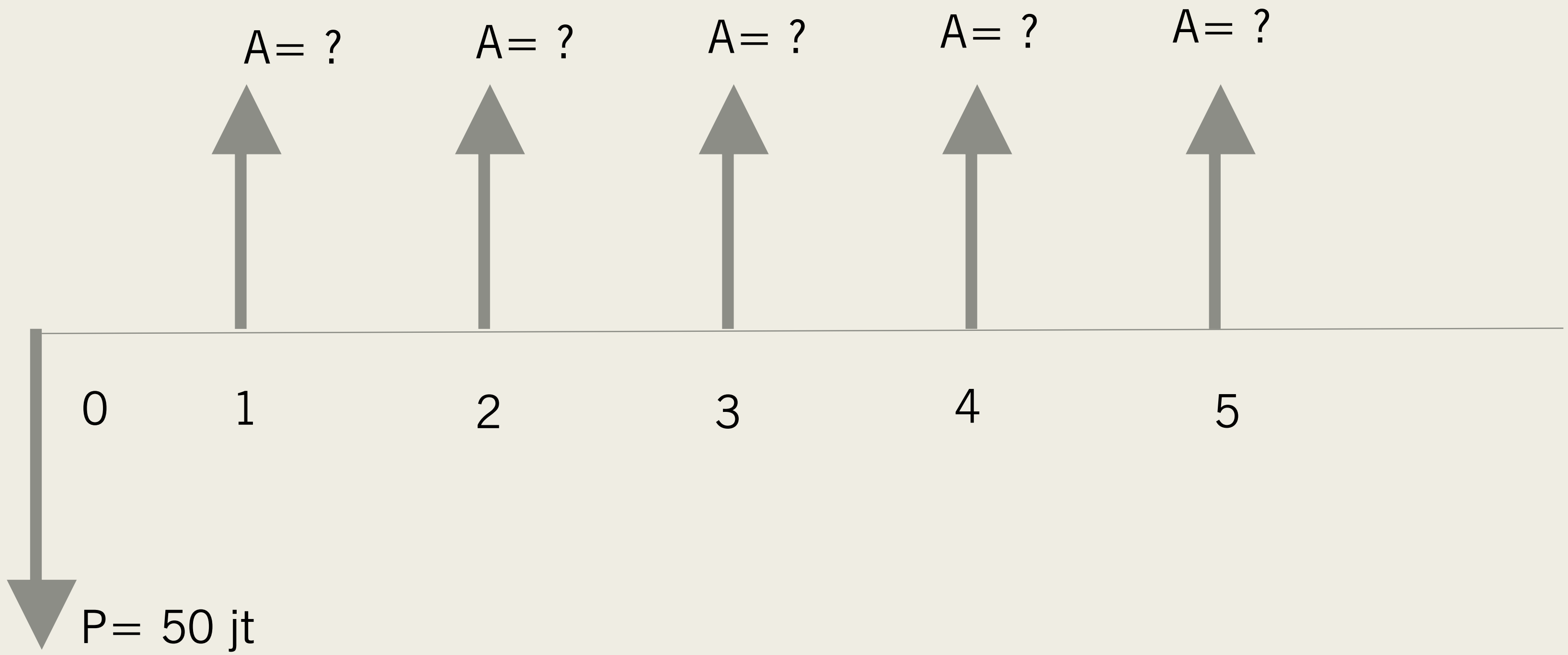
$$\left(\frac{(1+i)^n - 1}{i.(1+i)^n}\right)$$: *uniform series present worth factor*

Practice Problem

An energy efficient machine cost Rp. 50.000.000 and has a life of 5 years. If the interest rate is 8%, how much must be saved every year to recover the cost of the capital invested in it?



Practice Problem



P= 50 jt, n= 5 years, A= unknown, I (APR)= 8%

A= P (A/P, I, 5)→ A= 50 (0,2505)= 12.520.000

Excel Formula

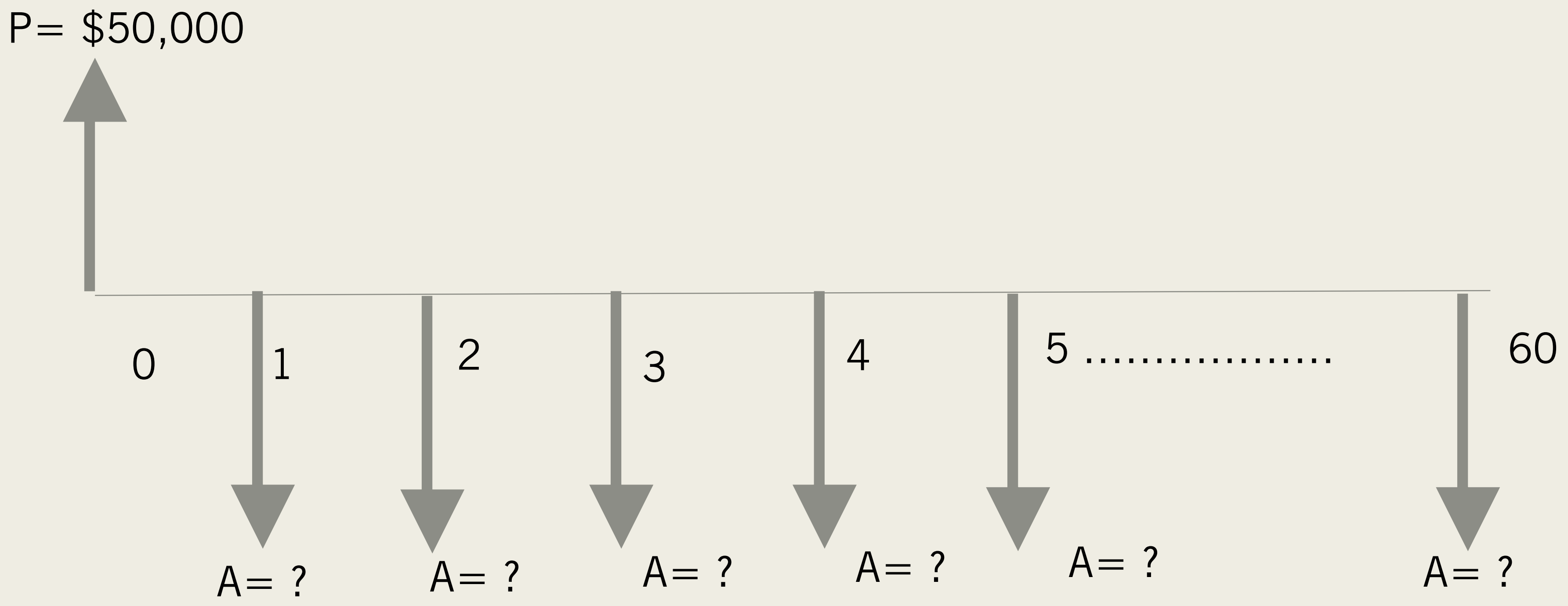
- The syntax of the function is: PMT(rate, nper, pv, [fv], [type])

rate	The interest rate, per period.
nper	The number of periods over which the loan or investment is to be paid.
pv	The present value of the loan / investment.
[fv]	An optional argument that specifies the future value of the loan / investment, at the end of nper payments. If omitted, [fv] has the default value of 0.
[type]	0 - the payment is made at the end of the period; 1 - the payment is made at the beginning of the period.

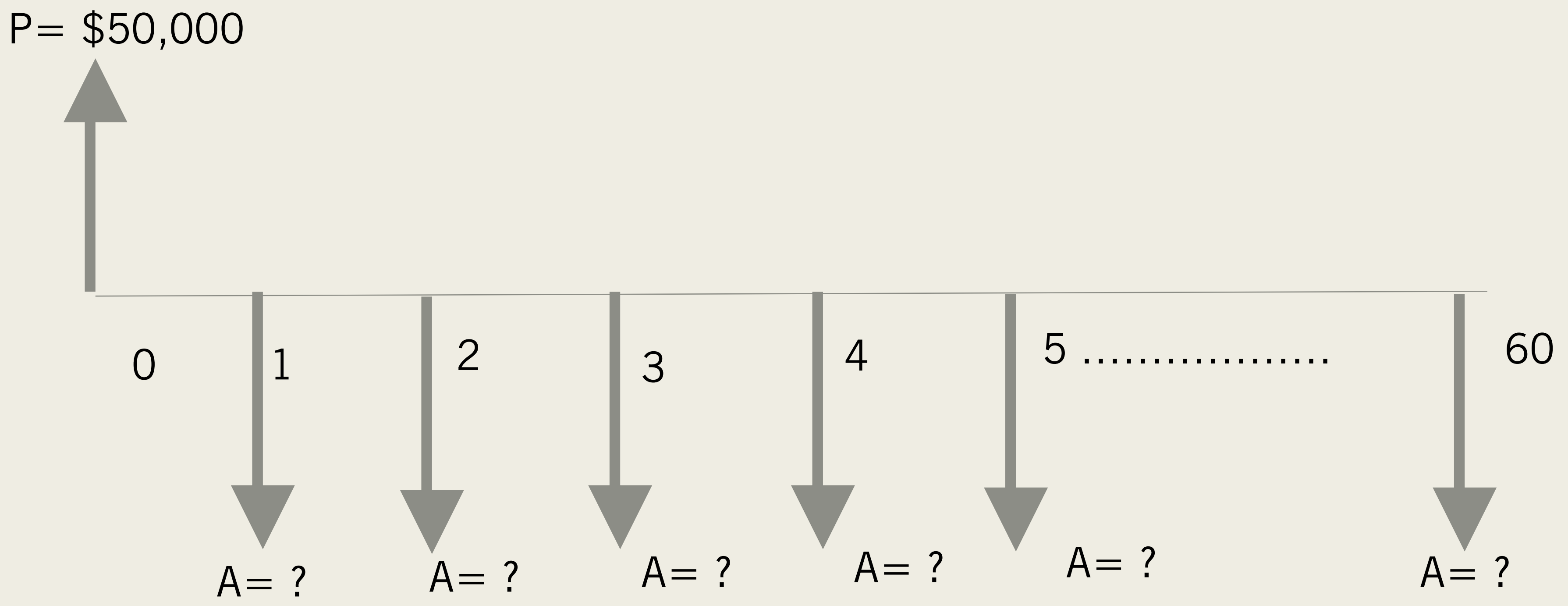
- =PMT(8%, 5, -50.000.000, 0,0) = 12.520.000

Practice Problem

- Calculate **the monthly payments** on a loan of $50,000 which is to be paid off in full after 5 years. Interest is charged **at a rate of 6% per year** and the payment to the loan is to be made at the end of each month.



Answer



P= $50,000 , n= 5 years, A= unknown, I (APR)= 6%

Monthly payment = 6%/12= 0,005, n=12x 5 years= 60 months

A= P (A/P, 0,005, 60) → A= $50,000 (0.0193)=965 (minus)

=PMT(6%/12,60,50000,0,0) = -966,64

Practice Problem

Father saves his salary up to Rp1.000.000 every month at the commercial bank that pays 2% monthly interest. Estimate his account at the end of year 3!

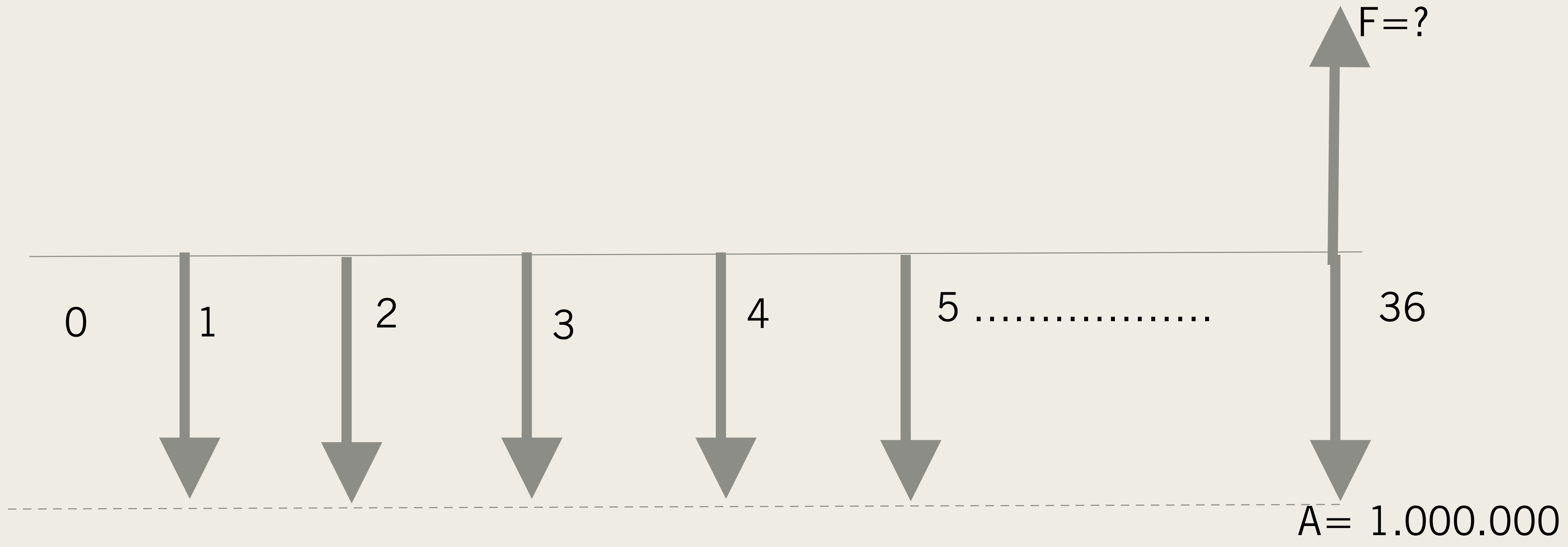
Argument:

A= Rp.1.000.000

N= 3 years= 36 months

I=2 %

Question : F ?



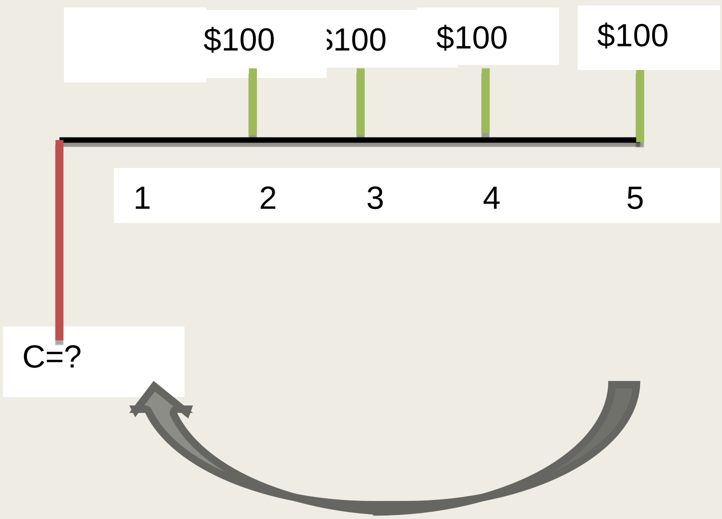
- Uniforms series compound amount factor for i= 2% equals to 51.994
- F= A(F/A, i, n) or 1.000.000 (F/A, 2%, 36)
- F=1.000.000 x 51.994
- F= 51.994.000

Excel formula *(see previous chapter)*

- =FV (rate, nper, pmt, [pv], [type])
  - ***rate*** *- The interest rate per period.*
  - ***nper*** *- The total number of payment periods.*
  - ***pmt*** *- The payment made each period.* ***Must be entered as a negative number***
  - ***pv*** *- The present value of future payments.* ***Not relevant in this case***
  - ***type*** *- [optional] When payments are due.* ***0 = end of period,*** *1 = beginning of period. Default is 0.*

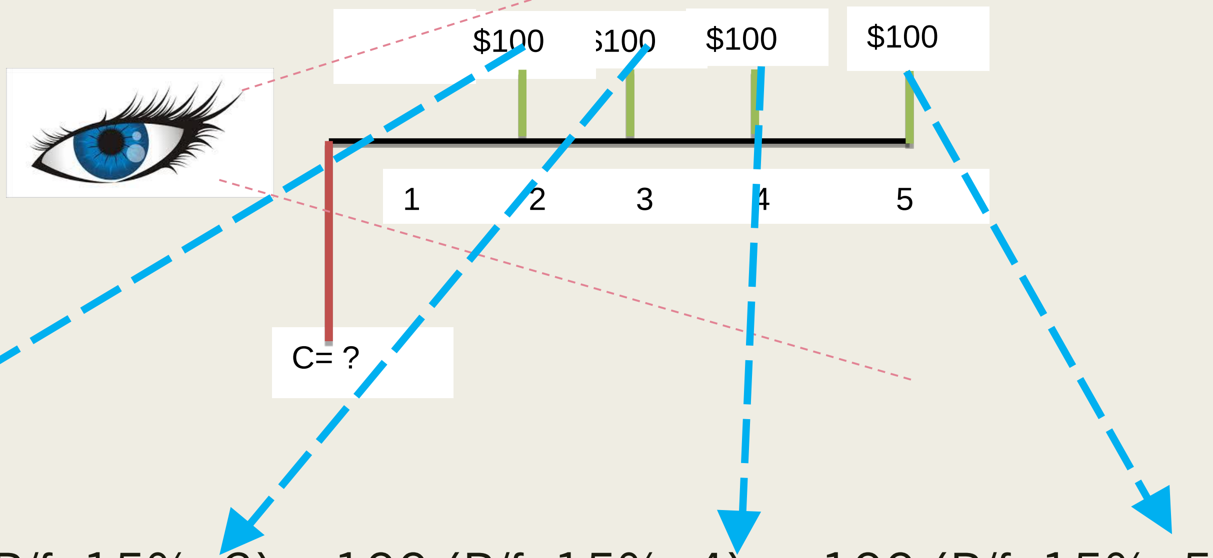
Excel Formula:
F = FV(2%,36,-1000000,0,0)
F= $51,994,367.19

Practice Problem



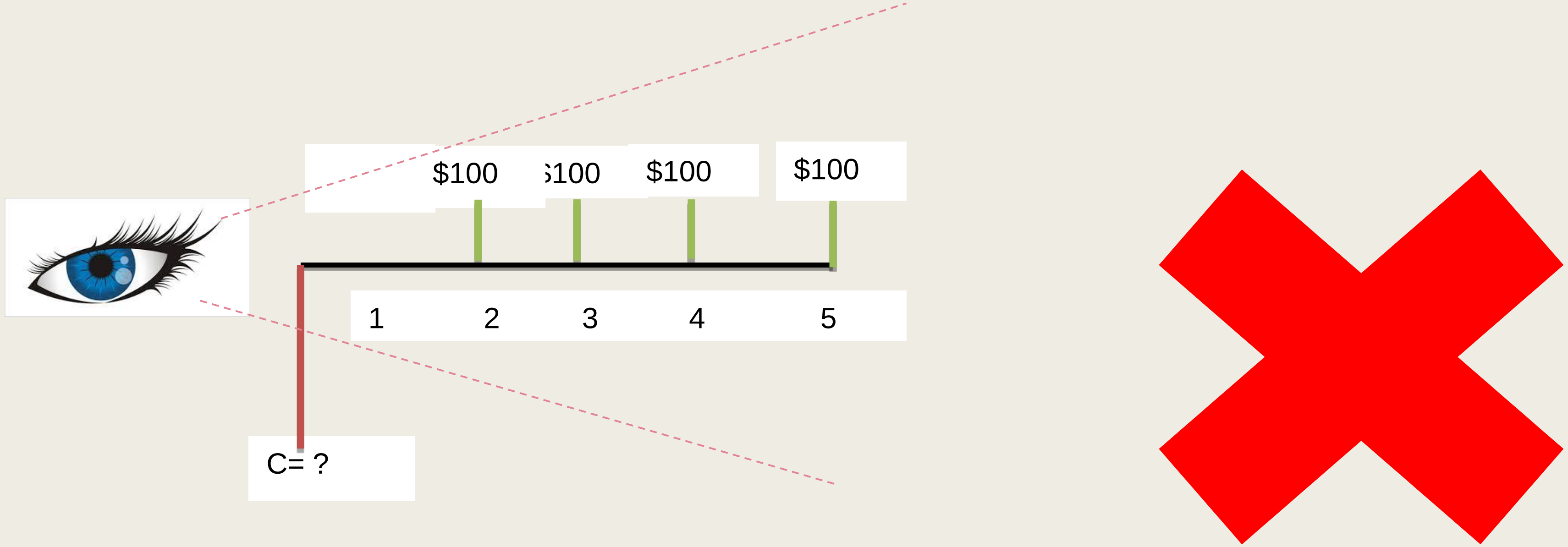
- i = 15%, calculate C!
- There are 2 ways to estimate C

First Way: one by one convert to present



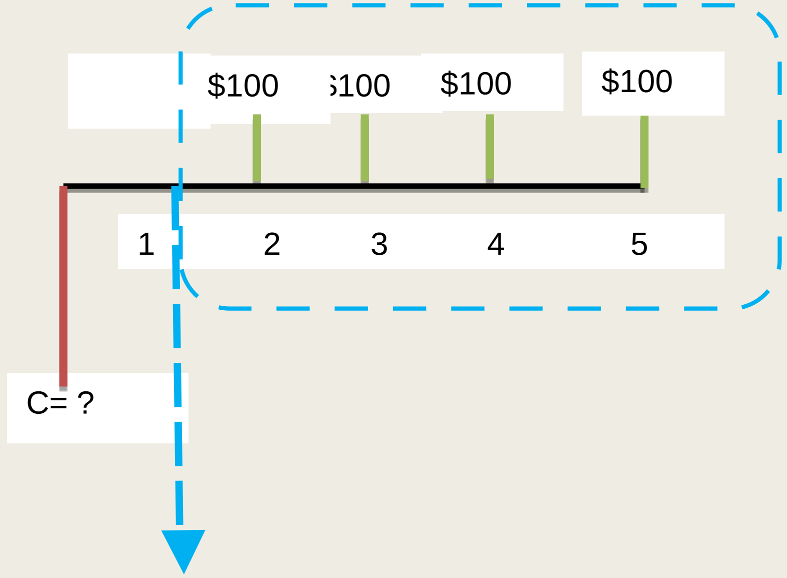
C= 100 (P/f, 15%, 2) + 100 (P/f, 15%, 3) +100 (P/f, 15%, 4) + 100 (P/f, 15%, 5)=…

Hold on! Can we use this answer?



$$C=A(P/A,\ 15\%,4)$$

The equivalence for annual cash flow converted to present always falls **one period earlier**

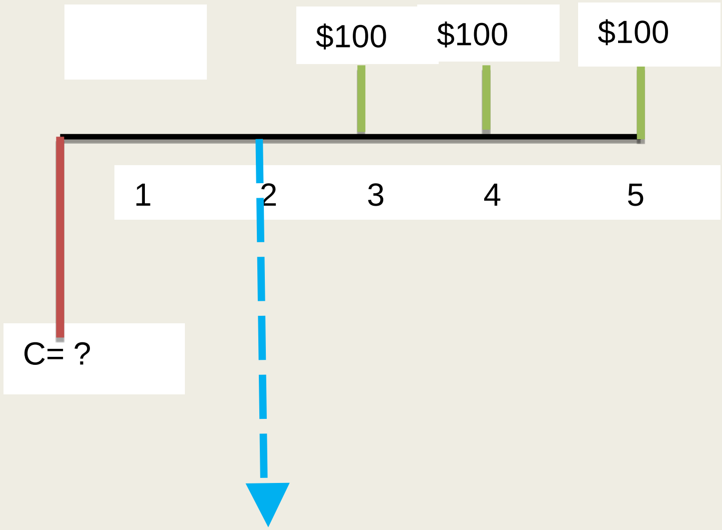


This is the estimate equivalence

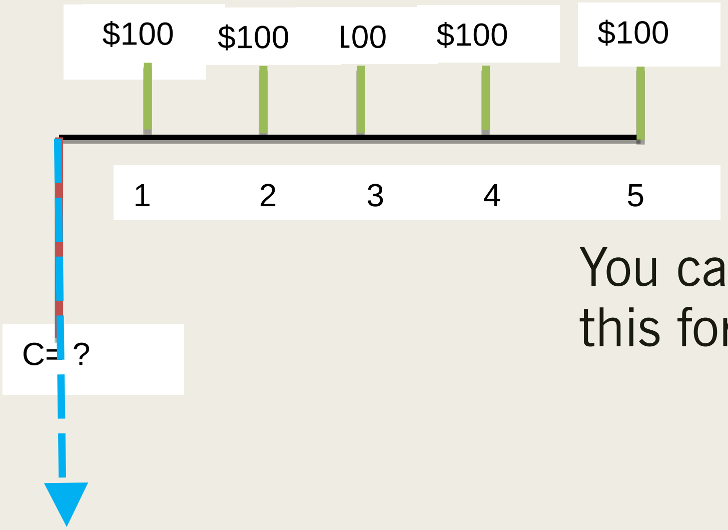
So this answer is not finish yet →C=A(P/A, 15%,4)

Because we want to find at zero period not the first period

So let see the example various cash flow diagrams below

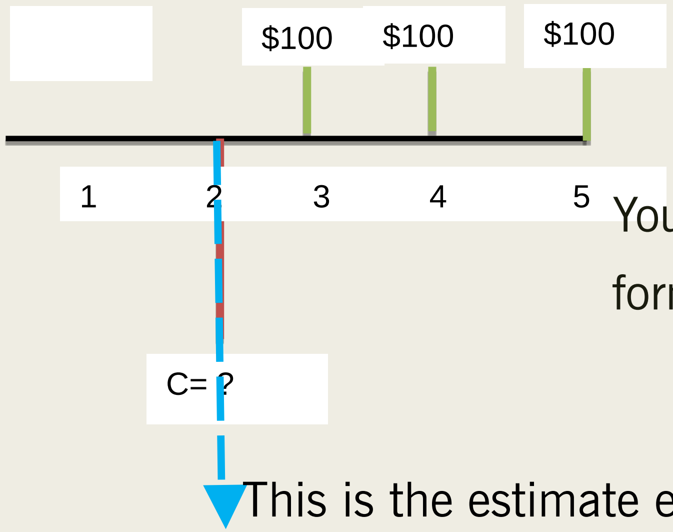


This is the estimate equivalence



You can solve directly using this formula C=A(P/A, 15%,5)

This is the estimate equivalence



You can solve directly using this formula C=A(P/A, 15%,3)

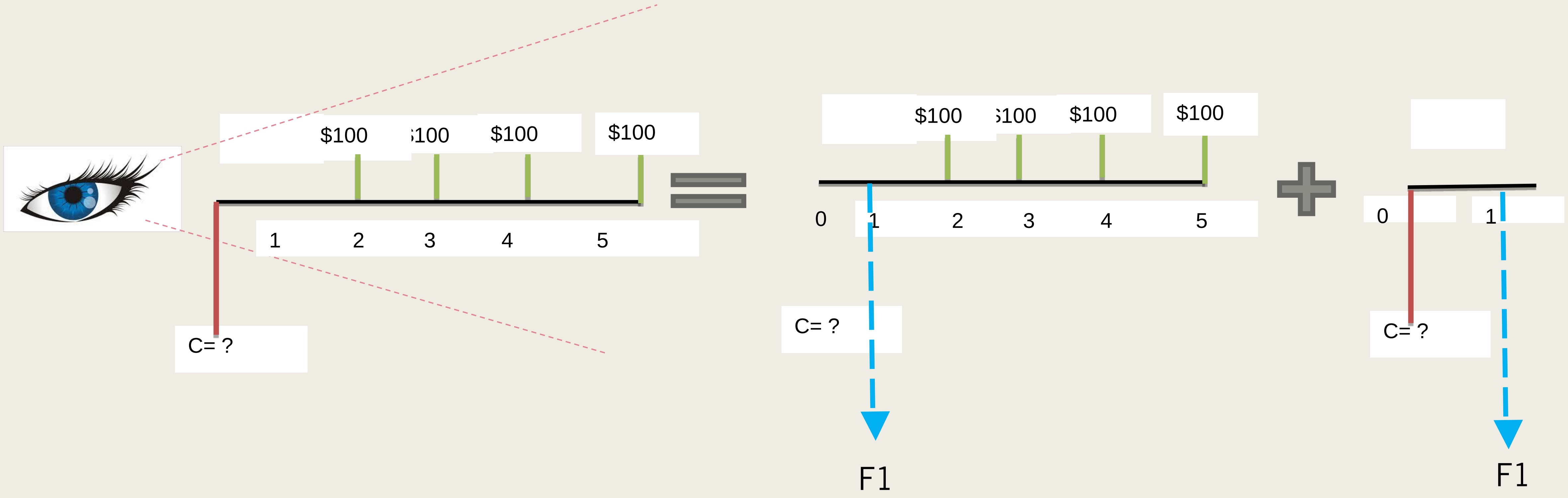
This is the estimate equivalence

You can solve directly using this formula **P=A(P/A,I,n)**

If only **the equivalence falls one period earlier**

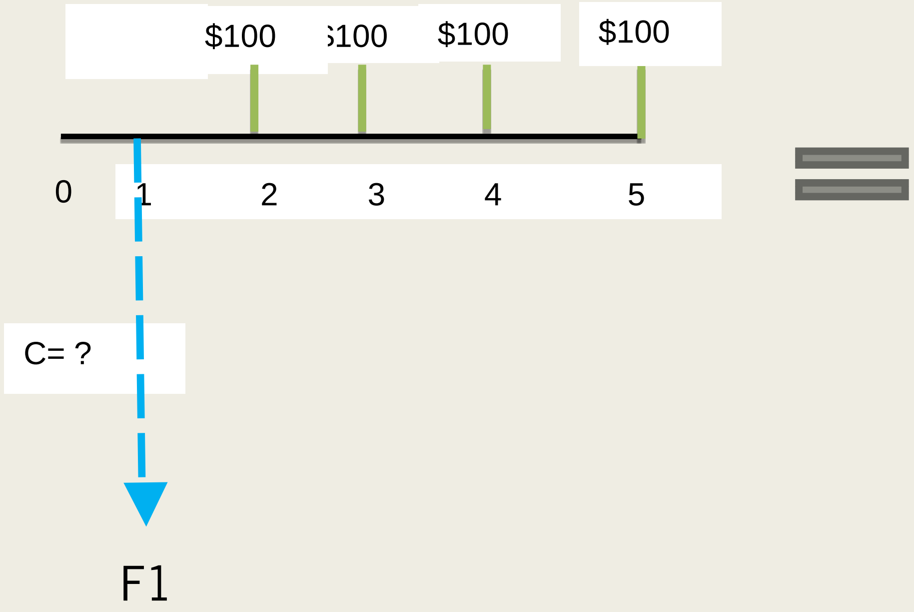
Practice Problem

Second Way: find the equivalence using present and annual relationship



Practice Problem

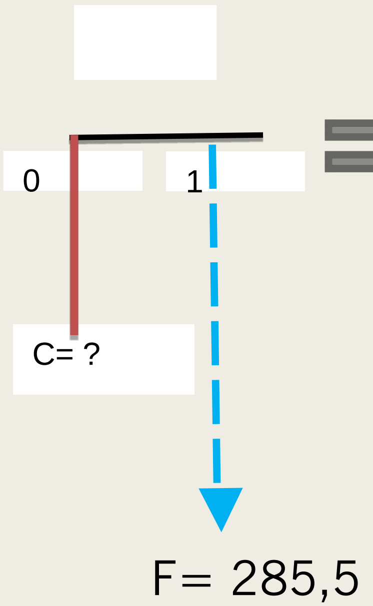
Second Way: find the equivalence using present and annual relationship



C=100 (P/A, 15%,4)= 100 (2.855)= 285,5

Practice Problem

Since the estimate equivalence is 285,5 which is the future of C
Then find the equivalence using present-future relationship



$$= \quad C=285,5(P/F,\ 15\%,1)=\ 285,5\ (0.8696)=\ 248,27$$

Excel formula *(see previous chapter)*

Syntax : =PV (rate, nper, pmt, [fv], [type])

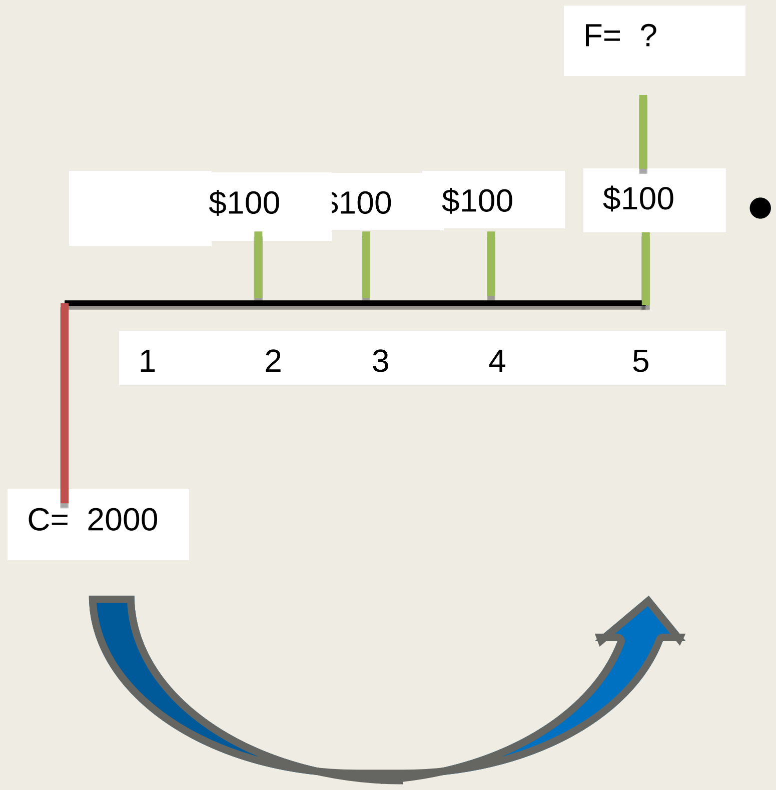
- **rate** - The interest rate per period.
- **nper** - The total number of payment periods.
- **pmt** - The payment made each period or any kind of earnings
- **fv** - [optional] A cash balance you want to attain after the last payment is made. If omitted, assumed to be zero.
- **type** - [optional] When payments are due. 0 = end of period, 1 = beginning of period. Default is 0.

Excel Formula:
=PV(15%,5,100,0,0)
=$285.22

→

Excel Formula:
=PV(15%,1,0,285.22,0)
=$248,2

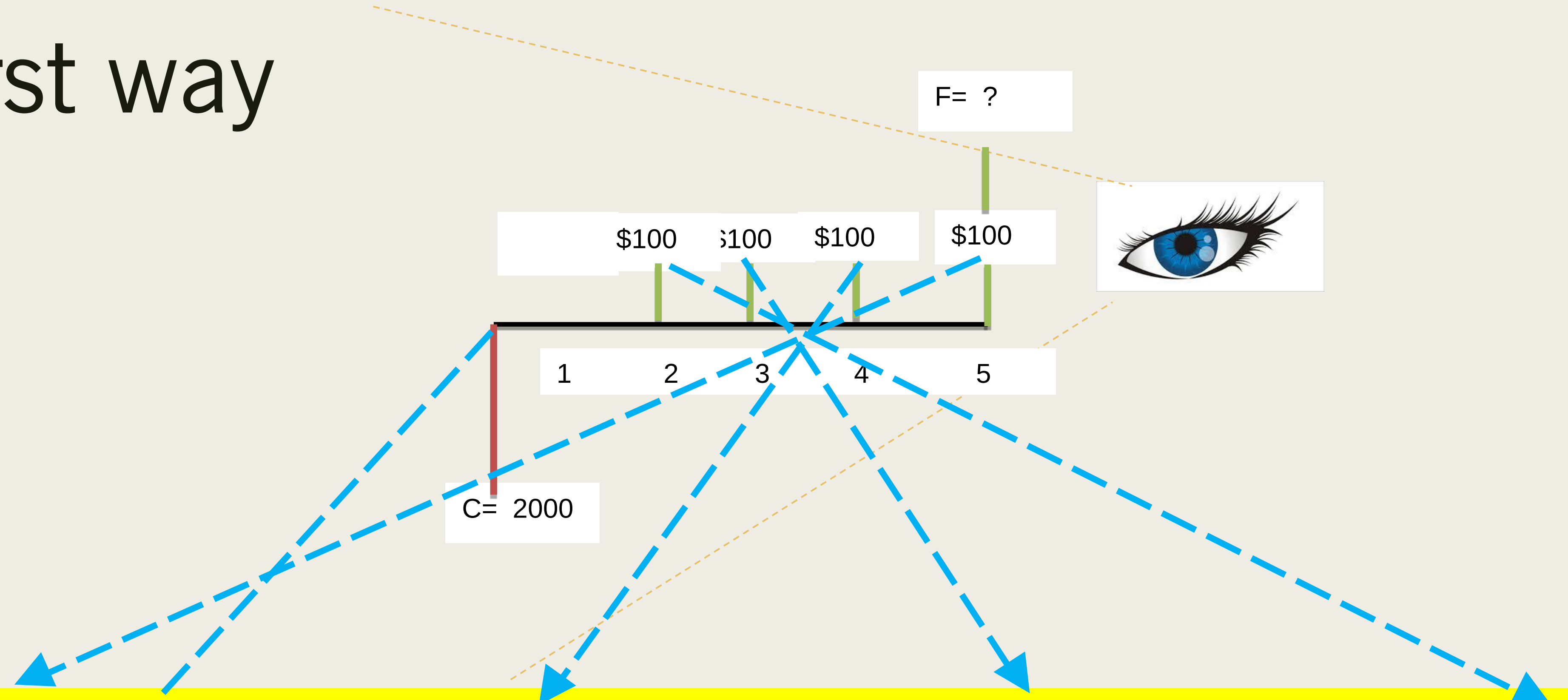
Relax, take a breath ☺



- i = 15%, calculate F!

There are also two ways to solve this problem

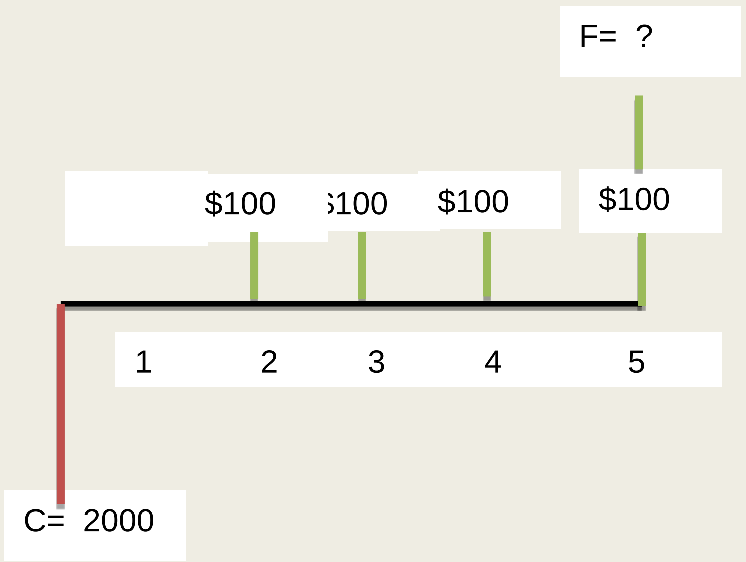
First way



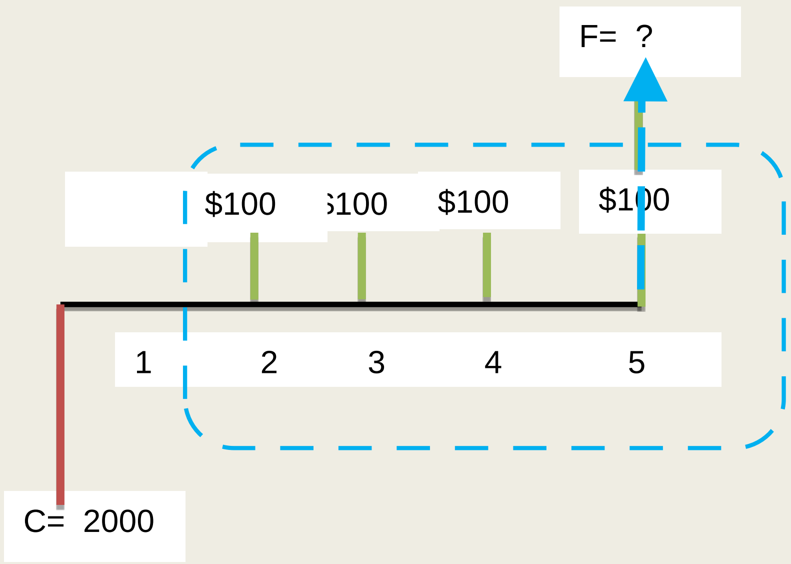
F= 100 + 100 (F/p, 15%, 1) +100 (F/p, 15%, 2) + 100 (F/P, 15%, 3)

- 2000 (F/p. 15%, 5)=…

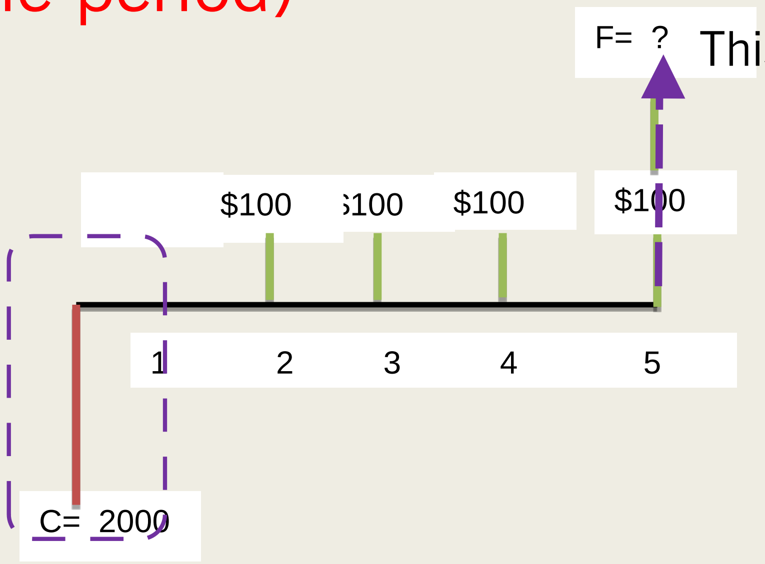
Second way



This is the estimate equivalence

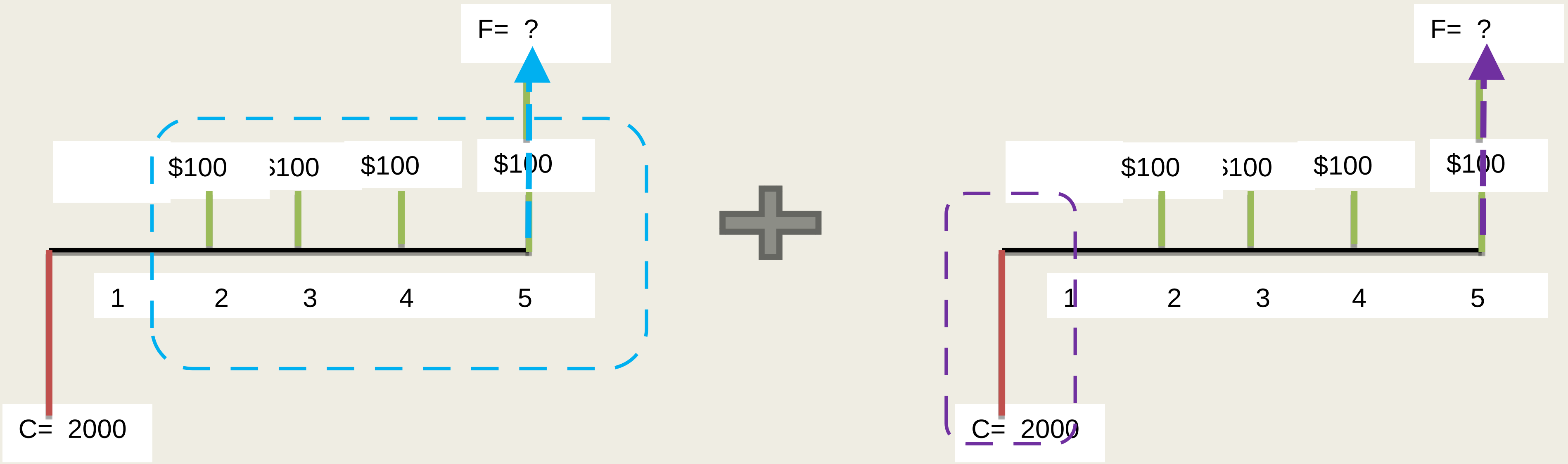


- The equivalence for annual converted to future falls at the latest cash flow (the same period)



This is the estimate equivalence

Second way

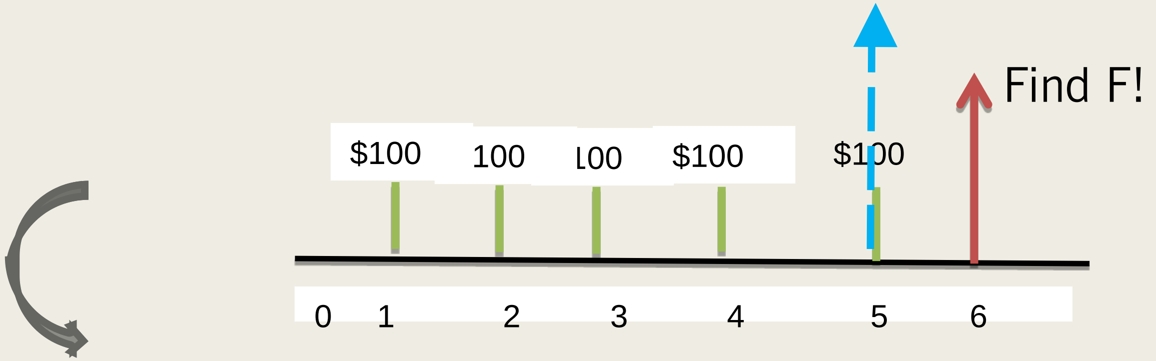


100 (F/A 15%, 4)=100 (4.993)= 499,3

-2000(F/P 15%, 5)=-2000 (2.011)

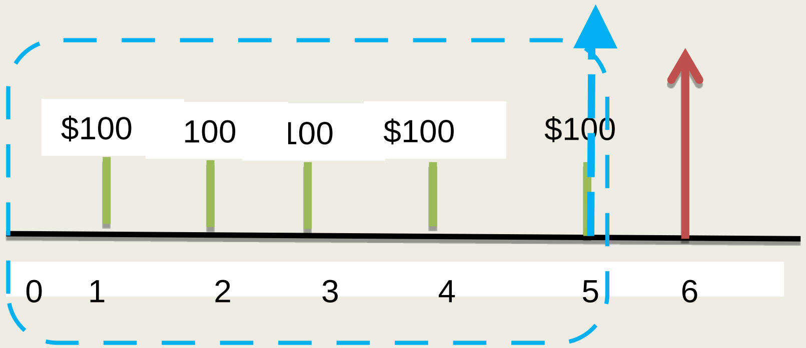
F= 499,3-4022= -3522,7

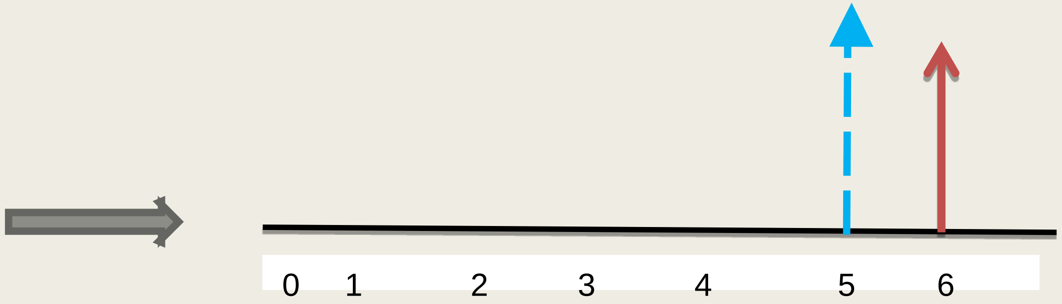
This is the estimate equivalence



F=100 (F/A,15%,5)= 100 (6.742)= 674,2

F1= 674,2





F= 674,2 (F/P. 15%, 1)=674,2 (1.150)=775,33

You can solve directly using this formula

F=A(F/A,I,n)

If only **the equivalence falls in the latest cash flow at the same time**

Practice Problem

- Consider the following problem P= \$6800, A= \$140, n=60, i=unknown
- Find the monthly interest rate!

P= A (P/A, I, n)

6800= 140 (P/A, I, 60)→ (P/A, I, 60)= 48,571

Look through **compound interest table** to find the values of (P/A, I, 60) that are **CLOSE** to 48,571

Interest rate	(P/A, I, 60)
½% →	51,726
i? →	48,571
¾% →	48,174

The interest formulas **are not linear,** so interpolation should be computed with **interest rate as close to the correct answer as possible**

Practice Problem

Interest rate	(P/A, I, 60)
X1= ½%	Y1= 51,726
X= i=?	Y= 48,571
X2= ¾%	Y2= 48,174

$$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1) \text{ interpolation formula}$$

$$\frac{48,571-\mathbf{51,726}}{48,174-\mathbf{51,726}} = \frac{x-0,5}{0,75-0,5}$$

$$\frac{-3,155}{-3,522} = \frac{x-0,5}{0,25}$$

0,78875 = 3,552x -1,776

i= 0,72% per month

Excel formula

- RATE(nper, pmt, pv, [fv], [type], [guess])

nper	The number of periods over which the loan or investment is to be paid.
pmt	The (fixed) payment amount per period.
pv	The present value of the loan / investment.
[fv]	An optional argument that specifies the future value of the loan / investment, at the end of nper payments. If omitted, [fv] takes on the default value of 0.
[type]	0 - the payment is made at the end of the period; 1 - the payment is made at the beginning of the period.
[guess]	An initial estimate at what the rate will be

Excel formula

- RATE(nper, pmt, pv, [fv], [type], [guess])

Interest rate	(P/A, I, 60)
½%	51,726
i?	48,571
¾%	48,174

P= \$6800, A= \$140, n=60, i=unknown

6800= 140 (P/A, I, 60)→ (P/A, I, 60)= 48,571

=RATE(60,-140, 6800, 0, 0, 0.25%) = 0,721% monthly

Annual rate of interest= 0,721 x 12 months= 8,65%

- F= A(F/A, i, n) or 1.000.000 (F/A, 2%, 36)

$$\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots . (1) \text{ interpolation formula}$$

Period	(F/A, 2%, n)
35	(F/A, 2%, 35)= 49,994
36	?
40	(F/A, 2%, 40)= 60,402

X1 y1

(F/A, 2%, 35)= 49,994

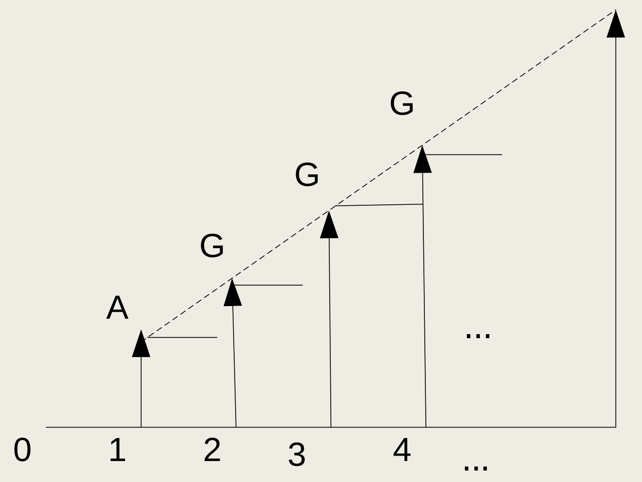
(F/A, 2%, 40)= 60,402

X2 y2

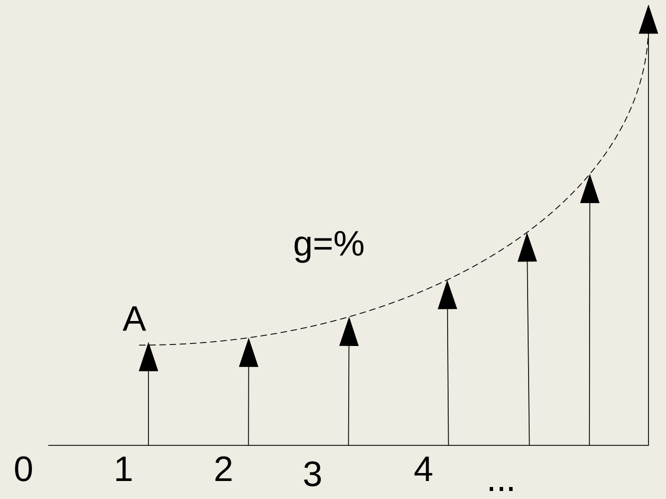
$$\frac{y-49{,}994}{60{,}402-49{,}994} = \frac{36-35}{40-35} \rightarrow \frac{y-49{,}994}{10{,}408} = \frac{1}{5}$$

y= 52, 0756

CASH FLOW GRADIENT



Pola Cash Flow
Arithmatic Gradient



Pola Cash Flow
Geometric Gradient

3. a) Cash Flow Arithmetic Gradient

$$F = G/i.\left(\frac{(1+i)^n - 1}{i} - n\right)$$ → **Not available in the table**

$$P = G.\left(\frac{(1+i)^n - in - 1}{i^2.(1+i)^n}\right)$$ → $P = G(P/G,i,n)$

$$\left(\frac{(1+i)^n - in - 1}{i^2.(1+i)^n}\right)$$: Arithmatic Gradient present worth factor

$$A = G.\left(\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right)$$ → $A = G(A/G,i,n)$

$$\left(\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right)$$: Arithmatic Gradient uniform series factor

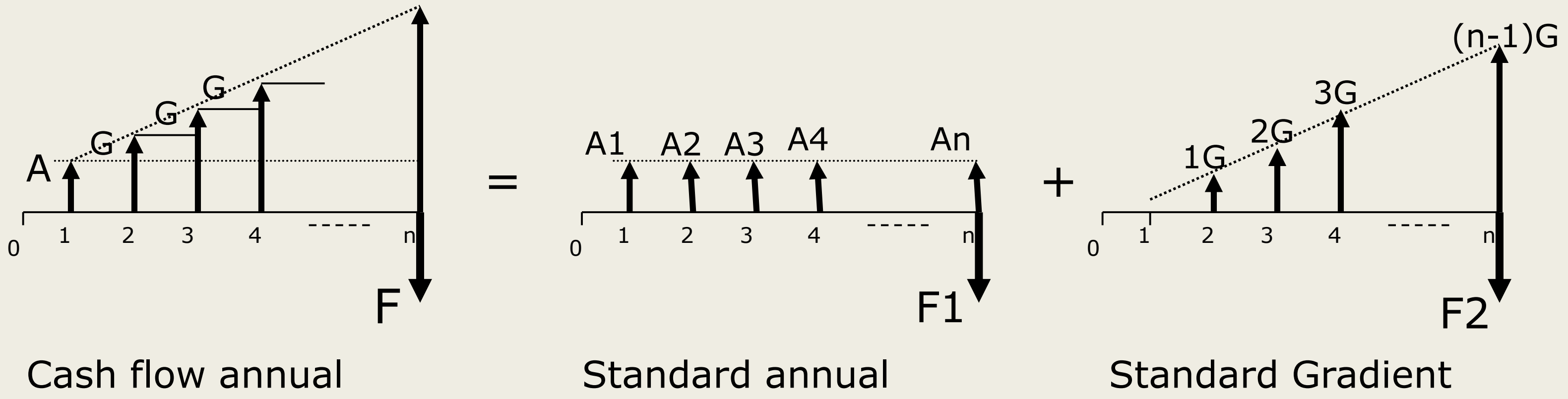
3. b) Cash Flow Geometric Gradient

$$P = A_1 \cdot \left(\frac{1-(1+g)^n \cdot (1+i)^{-n}}{1-g} \right)$$

While $i \neq g$

→ **Not available in the table**

Using 2 equations, standard annual and standard gradient



Practice Problem

- The shoes company in Cibaduyut has sold Rp.300.000.000/year shoes and want to gain more profit up to 50 million rupiah by marketing program.

  If the APR is 12 % then estimate:

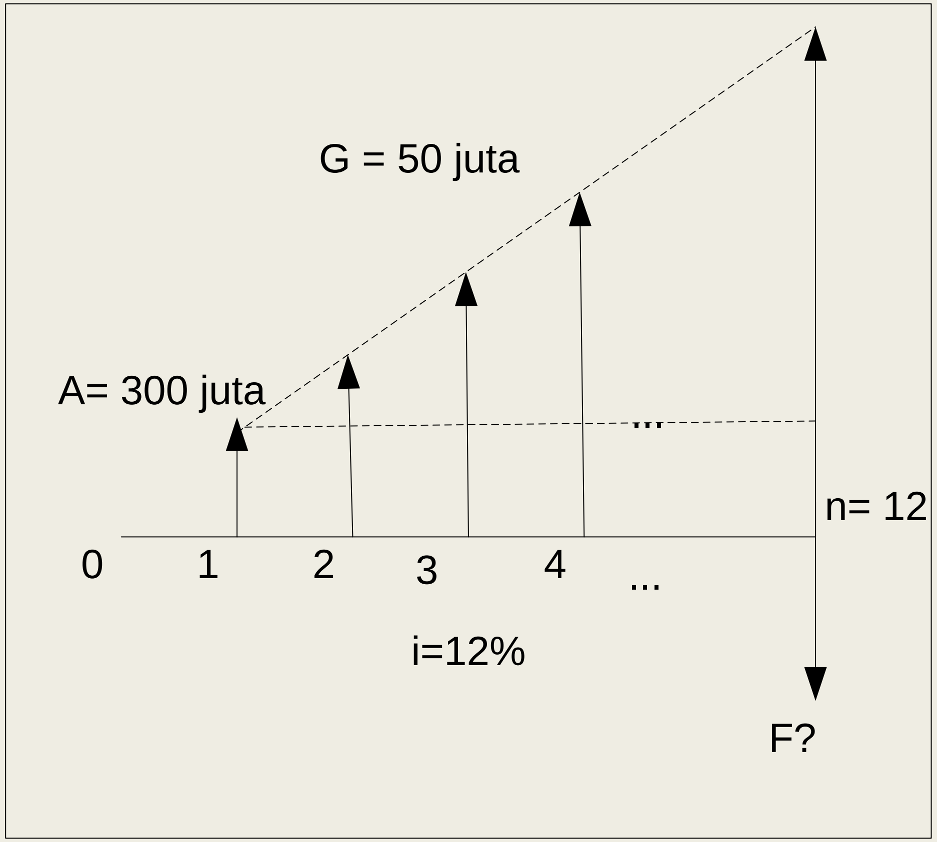
  – *Future equivalence*
  – *Present equivalence*

Argument

- A = 300 juta
- G=50 juta
- I= 12%

Question : F? and P?

answer



- F = $\frac{G}{i}\left[\frac{(1+i)^n}{i} - n\right]$

  to find the relationship between future and cash flow gradient

- F= $A\left[\frac{(1+i)^n - 1}{i}\right]$

  to find the relationship future and annual

  (available in the compound table F = A(F/A,i,n))

$$F = \frac{G}{i}\left[\frac{(1+i)^n}{i} - n\right] + A\ \left[\frac{(1+i)^n - 1}{i}\right]$$

$$F = \frac{50}{0{,}12}\left[\frac{(1+0{,}12)^{12}}{0{,}12} - 12\right] + 300\left[\frac{(1+0{,}12)^{12} - 1}{0{,}12}\right]$$

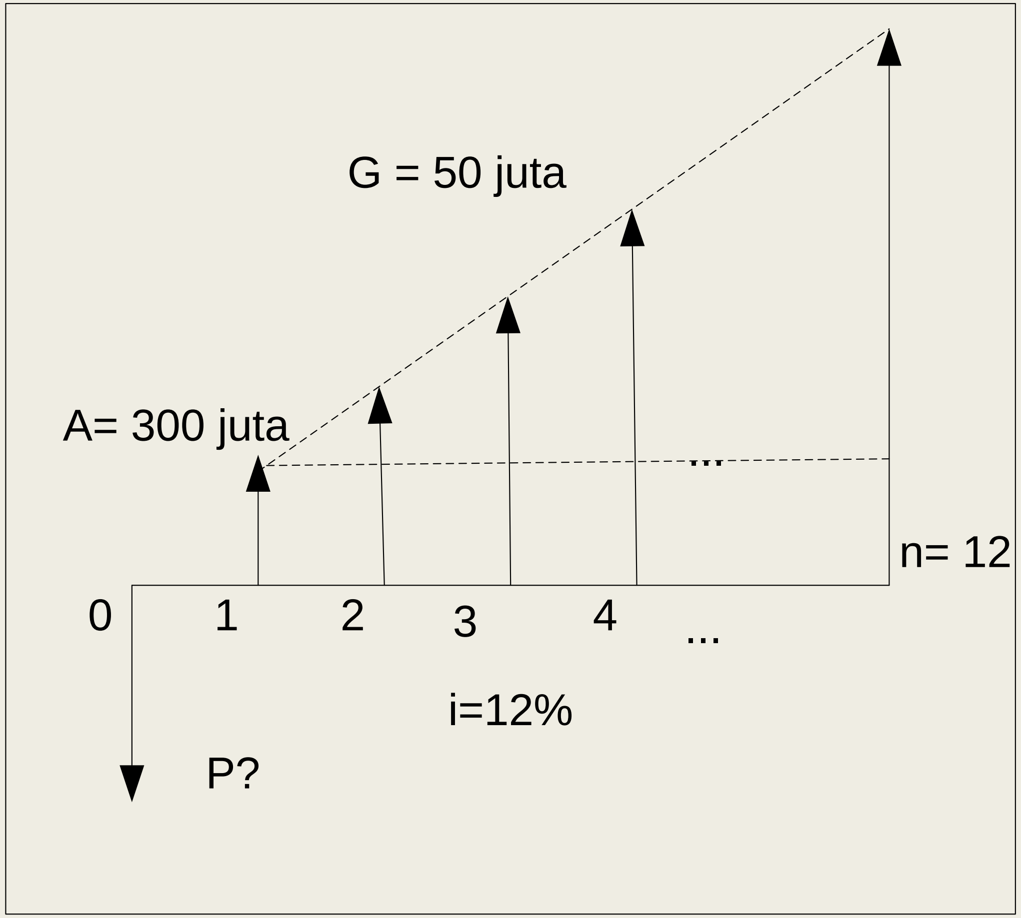
F= 416,66 (12,1333) + 300 (24,133)

F= 5055,472 + 7239,94

F= Rp. 12295,41

F= Rp12.295.412.178

P?



- P= G(P/G, i, n) + A(P/A,i,n)

  (both formula can be found in the compound interest table)

  P= 50 (25.952) + 300 (6.194)

  P= 1297,6 + 1858,2

  P= 3155,8