ENGINEERING ECONOMY

Nominal and Effective Interest Rates

Introduction

- If payments occur **more frequently than annual,** how do you calculate economic equivalence?
- **If interest period is other than annual,** how do you calculate economic equivalence?

Introduction

- However, interest can be managed in certain period for example: **monthly, quarterly, semiannually, etc**

12% Compounded **Monthly**

- What It Really Means?
  - Interest rate per month (i) = 12%/12 = 1%
  - Number of interest periods per year (N) = 12
- In words,
  - Bank will **charge 1% interest each month** on your unpaid balance, if you **borrowed money**
  - You will **earn 1% interest each month** on your remaining balance, if you **deposited money**

Nominal VS Effective

Nominal Interest Rate:

Interest rate quoted based on an **annual period**

Without considering effect of any compounding

Effective Interest Rate:

Actual interest earned or paid in a year or **some other time period**

Annual interest rate takin into account the effect of any compounding during the year

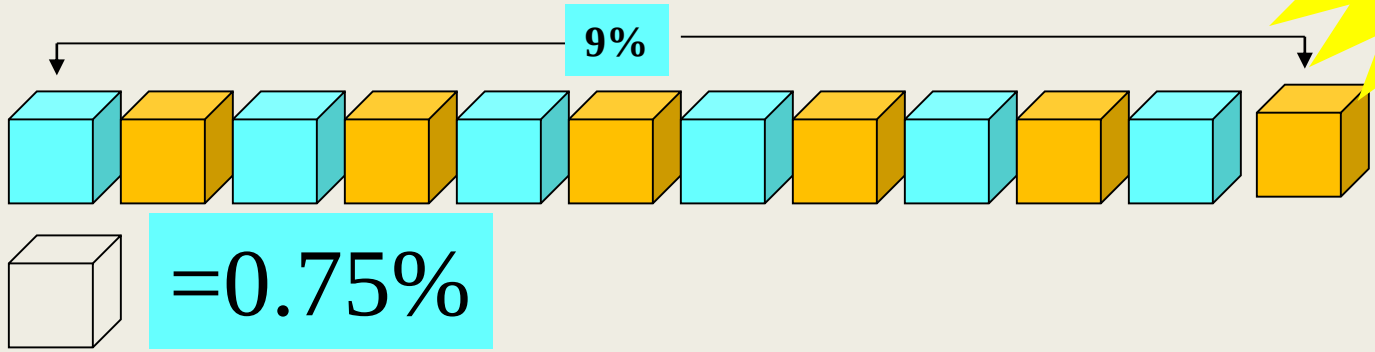
9% Compounded **Monthly**

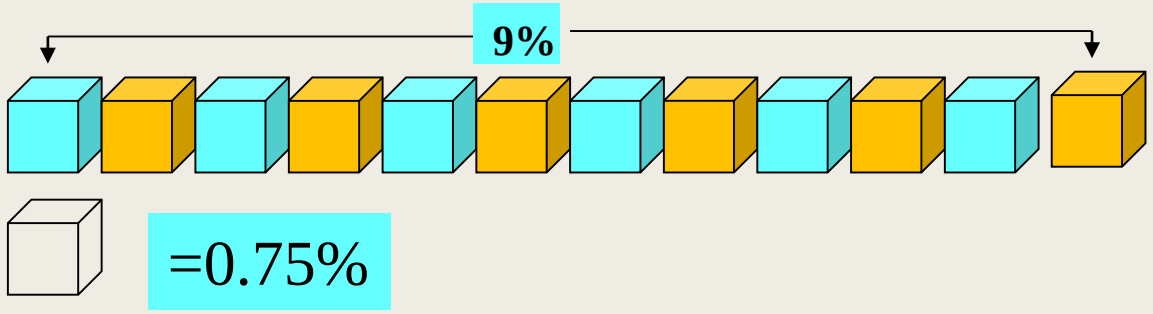
Question:
Suppose that you invest $1 for 1 year at 9% compounded monthly. How much interest would you earn?

Answer:
P= $1, n= 12, APR= 9%, monthly interest= 9%/12= 0,75%
F= P(F/P, I, n)
F= 1 (F/P, 0.75%, 12)
F= 1 (1.094)
F= 1,094

Interest= 1,094 -1= 0,094 or 9,4%

12 times earnings





9% compounded
monthly
or
0,75% per month
for **12 months**

=

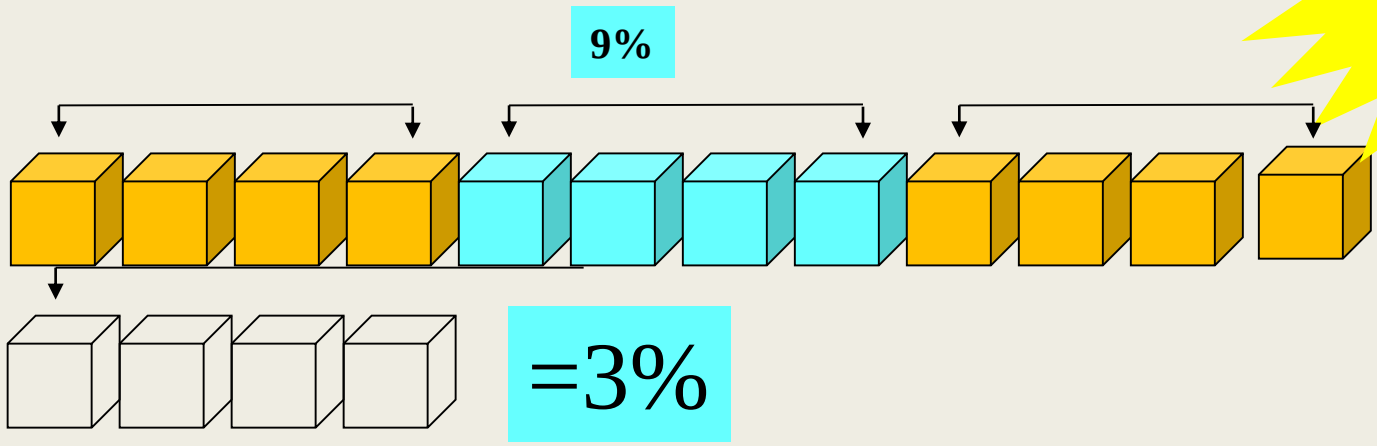
9,4 %
compounded
annually

9% Compounded **quarterly**

Question:
Suppose that you invest $1 for 1 year at 9% compounded quarterly. How much interest would you earn?

Answer:
P= $1, n= 12, APR= 9%, quarterly interest= 9%/3= 3% (every 4 months)
F= P(F/P, I, n)
F= 1 (F/P,3%, 3)
F= 1 (1.093)
F= 1,093

Interest= 1,093 -1= 0,093 or 9,3%



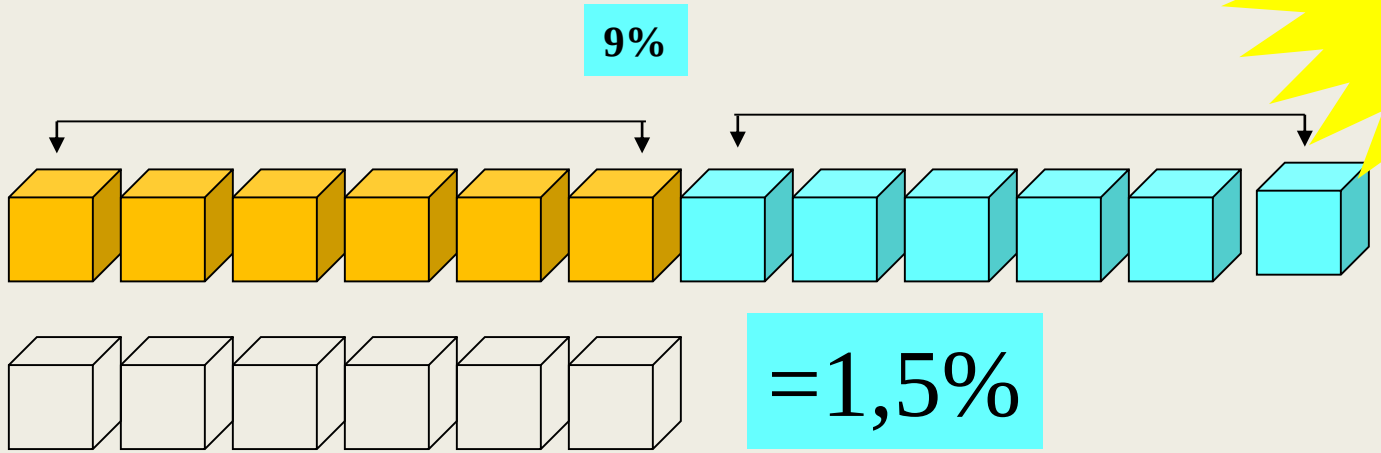
9% Compounded **Semiannually**

Question:
Suppose that you invest $1 for 1 year at 9% compounded semiannually. How much interest would you earn?

Answer:
P= $1, n= 12, APR= 9%, semiannually interest= 9% /2= 4,5% (every 6 months)
F= P(F/P, I, n)
F= 1 (F/P, 4.5%, 2)
F= 1 (1.092)
F= 1,092

Interest= 1,092 -1= 0,092 or 9,2%

2 times

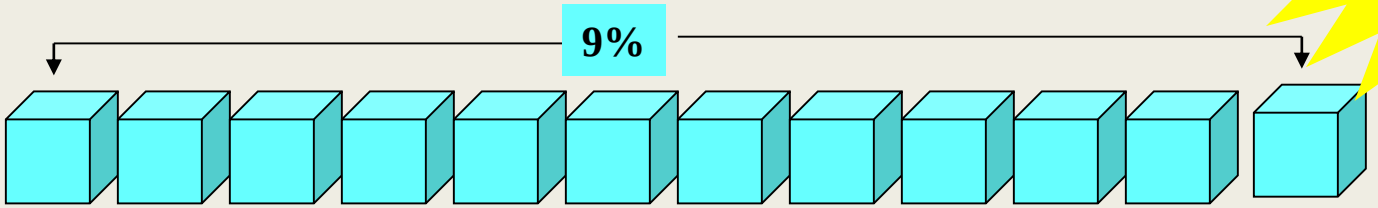


9% Compounded **annually**

Question:
Suppose that you invest $1 for 1 year at 9% compounded annually. How much interest would you earn?

Answer:
P= $1, n= 12, APR= 9%,
F= P(F/P, I, 1)
F= 1 (F/P, 9%, 1)
F= 1 (1.090)
F= 1,090

Interest= 1,090 -1= 0,092 or 9,0%



1 time

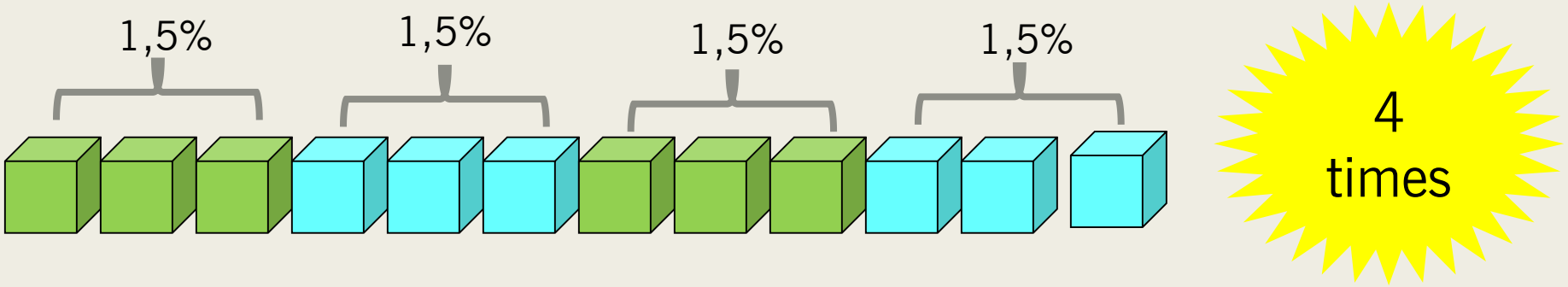
Effective Annual Interest Rate (Yield)

$$i_a = (1 + r / M)^M - 1 \quad \text{or} \quad i_a = (1 + i)^M - 1$$

r = nominal interest rate per year
i_a = effective annual interest rate
M = number of interest periods per year

Practice Problem

- If a savings bank pays **1,5% interest every 3 months,** what are the nominal and effective interest rate per year?
- Solution:



- Nominal interest rate per year: r= 4 x 1,5%= 6%

$$i_a = (1 + r/M)^M - 1$$

$$i_a = (1 + 0{,}06/4)^4 - 1 \quad = 6{,}1\%$$

Practice Problem

A loan shark lends money on the following terms:

"IF I GIVE YOU $50 ON MONDAY, YOU OWE ME $60 ON THE FOLLOWING MONDAY"

1. What **nominal** interest rate per year (r) is the loan shark charging?
2. What **effective** interest rate per year (i_d) is he charging?
3. If the loan shark started with $50 and was able to keep it, as well as the money he received, loaned out all the times, how much money did he have **at the end of one year**?

Solution for no. 1

Nominal interest rate per year

Argument:

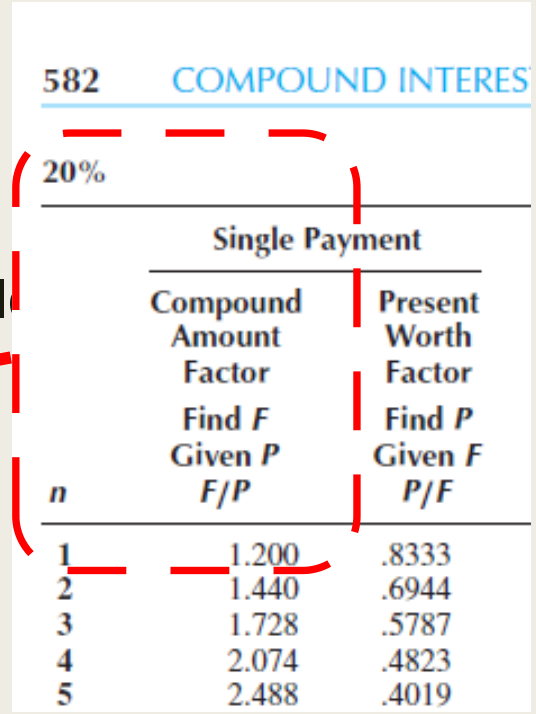
"IF I GIVE YOU $50 ON MONDAY, YOU OWE ME $60 ON THE FOLLOWING MONDAY"

P= $50, F= $60, n= 1 (week)

F= P(F/P, I, 1)

60 = 50 (F/P, I, 1) → **(F/P, I, 1)= 1,2** >>>>look through interest tabl

Therefore, i=20% per week



582 COMPOUND INTERES

20%

n	Single Payment: Compound Amount Factor Find F Given P F/P	Single Payment: Present Worth Factor Find P Given F P/F
1	1.200	.8333
2	1.440	.6944
3	1.728	.5787
4	2.074	.4823
5	2.488	.4019

Solution for no. 1 and 2

Nominal and effective interest rate per year

Nominal interest rate per year = 52 weeks x 0,20 = 10,4= **1040%**

Effective annual interest rate

$$i_a = (1 + r / M)^M - 1$$

$$i_a = (1 + 10{,}4 / 52)^{52} - 1$$

$$i_a = 13.105 - 1$$

$$i_a = 13.104 \quad = 1.310.400\%$$

Solution for no. 3
future value at the end of one year

From previous solution we get **i=20% per week**

The loan who start with $50 would get:

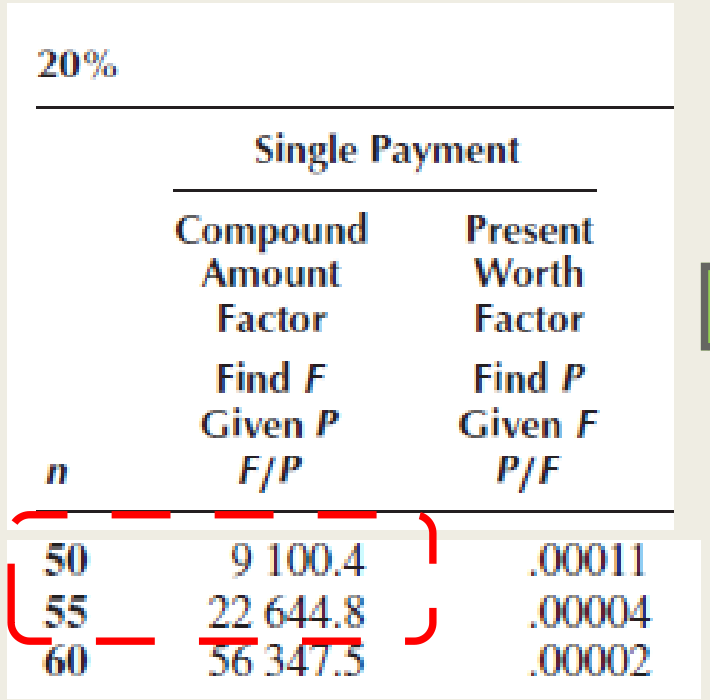
$$F = P\ (F/P, I, n) \text{ or } F = P\ (1+i)^n$$

$$F = 50\ (1+0{,}20)^{52}$$

$$F = \$655231{,}5$$

With Nominal interest rate per year **1040%** effective interest rate **1.310.400%** per year, the loan shark will get **$655,2** at the end of one year

You can also solve using **interpolation formula**
Compound amount factor for **n=52** is located between **n=50 to 55**



20%

n	Single Payment: Compound Amount Factor — Find *F* Given *P* *F/P*	Single Payment: Present Worth Factor — Find *P* Given *F* *P/F*
50	9 100.4	.00011
55	22 644.8	.00004
60	56 347.5	.00002

n	(F/P, I, n)
X1=50	Y1= 9100,4
X= 52	Y= ?
X2=55	Y2=22.644,8

n	(F/P, I, n)
X1=50	Y1= 9100,4
X= 52	Y= ?
X2=55	Y2=22.644,8

$$\frac{y-y1}{y2-y1}=\frac{x-x1}{x2-x1}$$

$$\frac{y-9100,4}{22.644,8-9100,4}=\frac{52-50}{55-50}$$

$$\frac{y-9100,4}{13.544,4}=\frac{2}{5}$$

$$5y-45.502=27.088,8$$

$$5y=72.590,8$$

$$y=14.518,16$$

F=$50 (F/P, 20%, 52)
F= $50 (14.518,16)
F= $725,908

- The more bigger gaps, the more error we get
- Interpolation is just approximation
- To find the future value in this case, using manual formula is recommended

Practice Problem

- If your credit card calculates the interest based on **12.5% APR**, what is your **monthly interest rate** and **annual effective interest rate**, respectively?
- Your current outstanding balance is $2,000 and **skips** payments for 2 months. What would be the total balance 2 months from now?

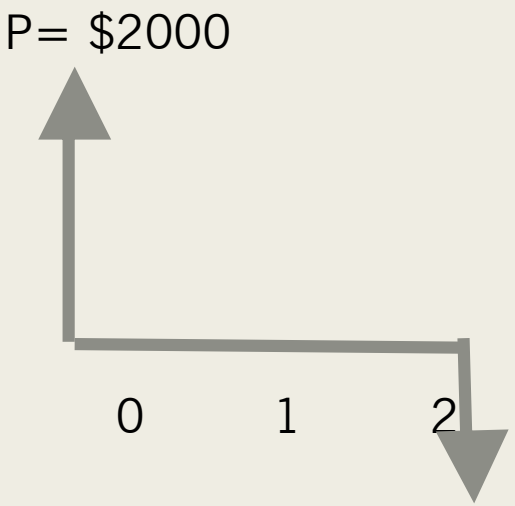
Solution

- Monthly interest rate= 12,5% / 12= 1,0417% per month
- Annual effective interest rate = 13,24%

$$i_a = (1+i)^M - 1$$

$$i_a = (1+0{,}010417)^{12} - 1$$

$$i_a = 0.132421$$



2 skips payment, total outstanding balance:

F=P (F/P, I, n)

F= 2000 (F/P, 1.0417%,2) >> **find in excel =FV (1.0417%, 2, 0, 2000, 0)**

F= ($2,041.89)