## ENGINEERING ECONOMY ANNUAL WORTH

## Annual Worth

- Change the present or future value into annual cashflow
- Select the largest Annual worth


## Annual Worth (Annual cash flow)

## Practice Problem

A lady purchased a copy machine for her office worth to $\$ 1000$. if she expects 10 years useful life from it, how much the annual value that equals to initial investment if the interest rate pays $7 \%$ ?

$$
\begin{aligned}
& \begin{array}{llllllllll} 
& & & & & & & & & \\
& \\
A & A & A & A & A & A & A & A & A
\end{array} \\
& P=\$ 1000 \quad n=10 \text { years } \\
& \text { i = } 7 \% \\
& =P(A / P, i, n) \\
& =1000(A / P, 7 \%, 10) \\
& =\$ 142,4
\end{aligned}
$$

Equivalent uniform annual cost
(EUAC)

## Annual Worth (Annual cash flow)

- If the residual value is already known, the amount of annual cash flow will be different

Practice Problem 2:
Refer to practice problem 1, calculate EUAC is the residual value is \$200 after 10 years usage?


- $E U A C=P(A / P i, n)-S(A / F, i, n)$
$=1000(A / P, 7 \%, 10)-200(A / F, 7 \%, 10)$
$=\$ 127,92$


## Annual Worth (Annual cash flow)

Since $(A / P, i, n)=(A / F, i, n)+i$, we can conclude from second practice problem:

EUAC $=P(A / P, i, n)-S(A / F, i, n)$
EUAC $=(\mathrm{P}-\mathrm{S})(\mathrm{A} / \mathrm{F}, \mathrm{i}, \mathrm{n})+\mathrm{Pi}$
$E \cup A C=(P-S)(A / P, i, n)+S i$

If the annual cost doesn't have the same number in every year, hence the EUAC calculation can use the following method:

1. Convert to Present worth with present worth factor: (P/F,i,n)
2. Then afterwards convert to annual worth (EUAC) using (A/P,i,n)

## Annual Worth (Annual cash flow)

## Practice Problem 3:

A car has useful life 5 years. It has maintenance and operational cost annually as follow:
year operational/maintenance cost
1
\$ 45
2 \$ 90
3 \$ 135
4 \$ 180
5 \$ 225
If the APR is $7 \%$, how much the EUAC?

## Annual Worth (Annual cash flow)

PW cost $=45(P / F, 7 \%, 1)+90(P / F, 7 \%, 2)+135(P / F, 7 \%, 3)+180(P / F, 7 \%, 4)+225(P / F, 7 \%, 5)$

$$
=\$ 531
$$

EUAC $=531(A / P, 7 \%, 5)=531(0,2439)$

$$
=\$ 129
$$

Or we can solve using gradient equivalence estimation:


- $\quad \mathrm{E} U A C=45+45(\mathrm{~A} / \mathrm{G}, 7 \%, 5)=45+45(1,865)$

$$
=\$ 129
$$

## Annual Worth (Annual cash flow)

From third practice problem we can summarize:

1. $\quad \mathrm{EUAC}=(\mathrm{PW} \operatorname{cost})(\mathrm{A} / \mathrm{P}, \mathrm{i}, \mathrm{n})$
2. Any kind expense or disbursement will increase EUAC value while any kind of earnings will decrease EUAC value
3. If the annual cost is deferent every year, then convert to present worth first and calculate to annual worth
4. If the annual cost increase the same number every year, you can use gradient equivalence

## Annual Worth (Annual cash flow)

| Fix input | Condition | Criteria |
| :---: | :--- | :--- |
| Fix Output | Amount of money or <br> input stays the same | maximize EUAB |
| Input \& Output are not fix | Profit or another fix <br> output | The amount of money <br> and other income, as <br> well as the advantages <br> and the other outputs are <br> not fixed | | maximize |
| :--- |
| (EUAB - EUAC) |

## Annual Worth (Annual cash flow)

- Example:
there are 3 alternatives to increase the productivity number in assembly line. Which alternative should be selected?

|  | Plan A | Plan B | Plan C |
| :--- | :---: | :---: | ---: |
| installment cost | $\$ 15000$ | $\$ 25000$ | $\$ 33000$ |
| Saving / years | 14000 | 9000 | 14000 |
| operational cost/ years | 8000 | 6000 | 6000 |
| Scrap value | 1500 | 2500 | 3300 |

## Annual Worth (Annual cash flow)

Since the installment cost and saving are not giving the same number, so the EUAC criteria is maximize (EUAB - EUAC)

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| Saving | \$ 14000 | \$ 9000 | \$ 14000 | \$ 0 |
| Scrap value(A/F,8\%,10) | 104 | 172 | 228 | 0 |
| EUAB = | \$ 14104 | \$ 9172 | \$ 14228 | \$ 0 |
| Installment cost (A/P, 8\%,10) | \$ 2235 | \$ 3725 | \$ 4917 | \$ 0 |
| Annual operational cost | 8000 | 6000 | 6000 | 0 |
| EUAC = | \$ 10235 | \$ 9725 | \$ 10917 | \$ 0 |
| $E \cup A B-E \cup A C=$ | \$ 3869 | \$ 553 | \$ 3311 | \$ 0 |

## Analysis Period

- The analysis period equal to the useful life of each alternative. These conditions are ideal conditions to be used in economic analysis
- The period of analysis is a multiple of the useful life. In this condition is necessary to replace one of the alternatives to the alternatives which has the same (identical) so that the same analysis period.


## Analysis Period for Annual Worth

- Pump A and Pump B has initial value \$ 7000 and $\$ 5000$ they also has residual value worth to $\$ 1500$ and $\$ 1000$, the useful life for the first pump is 12 years and for the second pump is 9 years. If the interest rate is $7 \%$, which pump should be purchased?


## Analysis Period

- Pump A (12 years)
$E \cup A C=(7000-1500)(A / P, 7 \%, 12)+1500(0,07)$

$$
=\$ 797
$$

Pump B (9 years)
$E \cup A C=(5000-1000)(A / P, 7 \%, 9)+1000(0,07)$

$$
=\$ 684
$$

EUAC B $<$ EUAC A, then select B

For different useful life, the least common multiple is not applied to annual worth concept

## Infinite Analysis Period

- The value of equivalent uniform annual cost (EUAC) concept can be used for infinite (n).
- The assumption is all of the cost and benefit consider to be constant. Based on this assumption EUAC for infinite analysis period) will be equal to EUAC for limited life $n$.

$$
\mathrm{EUAC}_{\substack{\text { for infinite } \\ \text { analysis period }}}=\mathrm{EUAC}_{\substack{\text { For limited } \\ \text { life } n}}
$$

## Infinite Analysis Period(2)

when $n=\infty$, we have $A=P i$ and, hence, $(A / P, i, \infty)$ equal i
$\mathrm{EUAC}_{\substack{\text { for infinite } \\ \text { analysis period }}}=P i+$ any other cost

Practice Problem:

The construction project of a city water channel development had two alternatives. Both of them was to build tunnel / tunnel through a mountain or building a pipeline that surrounded the mountain. The initial cost of the construction of the tunnel is \$5.5 million while the construction of the pipeline takes $\$ 5$ million for 50 years useful life. Which is the best alternative for building a water channel?

## Infinite Analysis Period(3)

■ Tunnel : permanent useful life

$$
(\mathrm{A} / \mathrm{P}, 6 \%, \quad)=\mathrm{i},
$$

$E \cup A C=P i=\$ 5,5$ million $(0,06)=\$ 330,000$
$\infty$

- Pipeline: 50 years useful life

EUAC $=\$ 5$ million (A/P,6\%,50)

$$
=\$ 5 \text { million }(0,0634)=\$ 317,000
$$

Select pipeline project construction

